Cartesian Grid Embedded Boundary Methods

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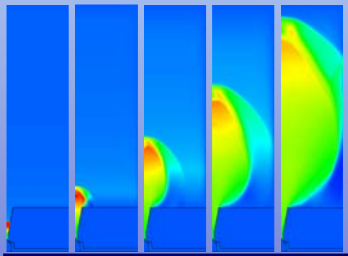
Embedded boundary methods are used to solve partial differential equations (PDEs) with complex geometries and internal interfaces. In this approach, the PDE is discretized on a regular Cartesian grid leading to high accuracy and efficiency, combined with low memory requirements. The most important advantage of an embedded boundary method is the straightforward generation of the computational mesh when compared to conforming unstructured grid approaches.

Plasma Wakefield Accelerator

A critical component in a plasma-based particle accelerator is the plasma waveguide generated by a gas jet passing through the nozzle and into the vacuum chamber. Simulation capabilities matching experimental data are essential in assisting with the design of the nozzle.



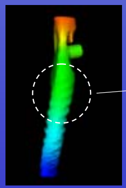
Time evolution of the expansion front of a gas jet in near vacuum conditions

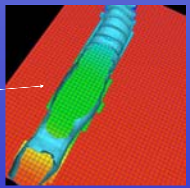
Biological Flows

Current techniques for surface extraction of medical images lose detail when meshed for simulations. Accurate methods are needed to extract surfaces without expense and loss in geometric detail, *and* to simulate the flows.

MR image of trachea

3D surface rendering





3D simulation on Cartesian mesh

Methods

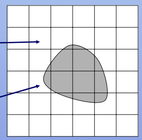
Embedded boundary grids are created by the intersection of the physical domain with the underlying Cartesian grid. The generation of the mesh is tractable, and coupling to mesh refinement techniques is straightforward.

<u>Cartesian Grid</u>

Standard finite difference discretization

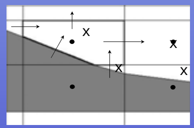
<u>Embedded Boundary</u>

Higher-order stencils in irregular, cut cells



Embedded Boundary Discretizations

Finite Volume Methods

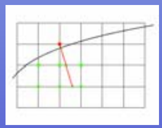


Conservation form of PDE is discretized using divergence theorem

$$\frac{\partial \vec{U}}{\partial t} + \nabla \cdot \vec{F}(\vec{U}) = \vec{S}$$

$$\nabla \cdot \vec{F}(\vec{U}) \approx \frac{1}{V}\int_V \nabla \cdot \vec{F}(\vec{U}) dV = \frac{1}{V}\oint_S \vec{F} \cdot \vec{n} dS$$

Finite Difference Methods

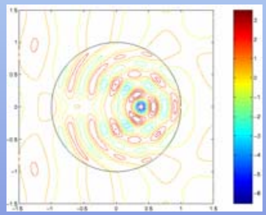


The surface normal is used to fulfill the boundary conditions needed to solve 2nd-order hyperbolic systems of PDEs

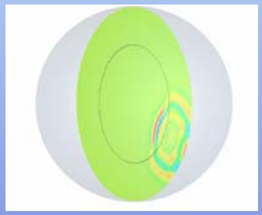
$$\frac{\partial^2 \vec{U}}{\partial t^2} = L\vec{U}$$

Seismic Wave Propagation

Seismic, acoustic and electromagnetic waves propagating in materials with discontinuously varying wave speeds and complex boundaries can be simulated in a straight forward manner.



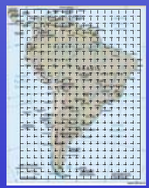
Electromagnetic wave in dielectric material with discontinuous permittivity



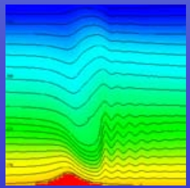
Seismic wave in sphere consisting of two different materials

Atmospheric Flows

Current simulations of global atmospheric flows use spatial steps that do not resolve accurately some of the effects generated by mountain ranges because of computational cost. Methods that allow for finer spatial steps over topography are needed.



Typical resolution



Gravity waves over a mountain

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